$\therefore AP=AR$ and $AP.a=AR.a$.

But $AP\sin A=b\sin C$ or $AP.a=bc=AR.a$.

$BP.a=(a-CP)a=a^2-CP.a=a^2-b^2$.

$CR.a=(a-BR)a=a^2-BR.a=a^2-c^2$.

$PR\sin A=AP\sin(\pi-2A)=2AP\sin A\cos A$.

$\therefore PR.a=2AP.a\cos A=2bc\cos A$.

(4) Since $BR.BC=AB^2$, $AB$ touches the circle through $ARC$ at $A$: therefore one of the Brocard points is on this circumference. Since $CP.CB=CA^2$, $CA$ touches the circle through $APB$ at $A$, which contains the other Brocard point.

(5) $BR=c^2/a$, $CR'=a^2/b$, $AR''=b^2/c$, $CP=b^2/a$, $BP'=a^2/c$, $AP''=c^2/b$.

$\therefore BR.CR'.AR''=CP.BP'.AP''=abc$; $(B'C')^2=c^2+b^3-2bc\cos 3A=a^2+8bc\cos A\sin^2 A$; $(A'C')^2=b^2+8ac\cos B\sin^2 B$, $(A'B')^2=c^2+8ab\cos C\sin^2 C$.

$$\therefore K'-K=8(bc\cos A\sin^2 A+ac\cos B\sin^2 B+ab\cos C\sin^2 C)$$
$$=32\triangle^2(\cos A/bc+\cos B/ac+\cos C/ab)$$
$$=(16\triangle^2/a^2b^2c^2)\Sigma(2a^2b^2-a^4)=256\triangle^4/a^2b^2c^2=16\triangle^2/R^2.$$

181. Proposed by **WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.**

Prove that the extremities of the latera recta of all ellipses having a given major axis $2a$ lie on the parabola $x^2=-a(y-a)$.

Solution by **L. C. WALKER, A. M., Petaluma High School, Petaluma, Cal.; J. R. HITT, Coral Institute, San Marcos, Tex.; and the PROPOSER.**

If $(x_1, y_1)$ be an extremity of one of the latera recta, plainly, $y_1=b^2/a$, or $b^2=ay_1$....(1); also, $a^2-b^2=a^2e^2=x_1^2$....(2), $b$ and $e$ having the usual meanings. Eliminating $b$ from (1) and (2), $x_1^2=-a(y_1-a)$.

Also solved by *J. SCHEFFER*, and *G. B. M. ZERR*.

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## CALCULUS.

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137. Proposed by **F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.**

Develop the equation of the curve assumed by the inextensible and revolving skipping rope.

No solution of this problem has been received.

138. Proposed by **M. E. GRABER, A. B., Tutor in Mathematics, Heidelberg University, Tiffin, O.**

Find the curve the length of whose arc measured from a given point is a mean proportional between the ordinate and twice the abscissa.

Solution by **G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa., and the PROPOSER.**

From the problem, $s^2=2xy$ or $s=\sqrt{(2xy)}$.

$$ds=\sqrt{(dx^2+dy^2)}=\frac{1}{\sqrt{2}}[\sqrt{(y/x)}dx+\sqrt{(x/y)}dy] \text{ or}$$

$$\sqrt{[1+(dy/dx)^2]}=\frac{1}{\sqrt{2}}[\sqrt{(y/x)}+\sqrt{(x/y)}(dy/dx)].$$

Let $y=mx$. $\therefore\ dy/dx=m+x(dm/dx)=m+px$.

$$\therefore\ \sqrt{[1+(m+px)^2]}=\frac{1}{\sqrt{2m}}(2m+px).\quad \frac{x^2(1-2m)p^2}{2m}+2x(1-m)p=(1-m)^2.$$

$$\therefore\ p=dm/dx=\frac{[1-m][1-\sqrt{(2m)}]\sqrt{(2m)}}{x[1-2m]}=\frac{\sqrt{[2m]}[1-m]}{x[1+\sqrt{(2m)}]}.$$

$$\therefore\ dx/x=\frac{[1+\sqrt{(2m)}]\,dm}{[1-m]\sqrt{[2m]}}.$$

$$\therefore\ \log[Cx(1-m)]=\frac{1}{\sqrt{2}}\log\left[\frac{1+\sqrt{m}}{1-\sqrt{m}}\right].$$

$$\log[C(x-y)]=\frac{1}{\sqrt{2}}\log\left[\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}}\right]=\frac{1}{\sqrt{2}}\log\left[\frac{x+y+2\sqrt{xy}}{x-y}\right].$$

For the given point $x=a$, $y=b$.

$$\therefore\ C=\frac{[a+b+2\sqrt{(ab)}]^{1/\sqrt{2}}}{[a-b][a-b]^{1/\sqrt{2}}}.$$

$$\therefore\ \frac{[x+y+2\sqrt{(xy)}]^{1/\sqrt{2}}}{[x-y][x-y]^{1/\sqrt{2}}}=\frac{[a+b+2\sqrt{(ab)}]^{1/\sqrt{2}}}{[a-b][a-b]^{1/\sqrt{2}}}.$$

$$\therefore\ \frac{a-b}{x-y}=\left[\frac{[x-y][a+b+2\sqrt{(ab)}]}{[a-b][x+y+2\sqrt{(xy)}]}\right]^{1/\sqrt{2}},\ \text{or}\ [y-x]^{\sqrt{2}}=c\,\frac{\sqrt{y}+\sqrt{x}}{\sqrt{y}-\sqrt{x}}.$$

**139. Proposed by WM. FRED FLEMING, Chicago, Ill.**

A tin watering-pot is constructed by joining the frustums of two right cones, so that their intersection is a mathematical one, their axes meeting at an angle of 45°. The bases of the smaller frustum are 2 inches and 4 inches in diameter, its altitude 8 inches. The bases of the larger frustum are 10 inches and 12 inches in diameter, its altitude 15 inches. In joining the two frustums the edges of the two larger bases are brought into coincidence. Water is poured into the vessel until it begins to run out of the spout. How many gallons (231 cubic inches) are required? How much water is in the spout and how much in the can? The vessel is tilted forward (in the plane of the axes of the two frustums) sufficiently to allow one-half of the water to run out. How much of the liquid is left in the spout and can, and what is the area of the surface of the water in spout and can? Through what angle has the vessel been tilted?

No solution of this problem has been received.